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Characterising of Impulse Radiating Antennas

INTRODUCTION

Impulse Radiating Antennas (IRA's) are increasingly of interest for various applications in UWB (Ultra Wideband) Radio, UWB-Sensing, the measurement of electrical emissions and High Power ElectroMagnetics (HPEM). IRA's are antennas which radiate over a large bandwidth and which have a short Impulse Response Function (IRF). The intention behind the use of IRA's is to distort the time shape of the received electromagnetic fields respectively the transmitted waveforms, as little as possible. This is significant for UWB Radio and HPEM in order to avoid the spreading of signal energy. For measurement and sensing purposes, there are no constraints by theory regarding the impulse behaviour of an antenna, only the bandwidth and their polarisation properties are of importance. Nevertheless IRA's are also preferred for these tasks. This avoids or reduces a spatial and temporal de-convolution of the antenna characteristic which would extremely burden the digital processing of measurement data.

The goal of this article is to introduce antenna characteristics which are of particular interest for UWB-sensing i.e. short range high resolution radar and positioning and to show some measurements. Some of the introduced parameters will be of general interest, while others are more or less important only for sensing purposes.

The classical approach to characterise antennas deals with the spectral domain i.e. sine wave excitation and power value measurements. Phase measurements are unusual, thus the impulse behaviour of the antenna remains unknown. To get complete information, phase measurements must be additionally introduced or the measurements have to be performed directly in the time domain. In what follows, the complex transmission formula (complex Friis equation) and its time domain counterpart will be derived in a simple intuitive way. Then the introduced characteristic antenna functions will be considered in more detail to get some hints for their measurement. Next, a measurement example will be shown including the measurement set-up and some results for a Vivaldi-antenna.

THE FRIIS-EQATION FOR TIME AND (COMPLEX) FREQUNCY DOMAIN

The following consideration will be performed in parallel for the frequency and time domain. The conversion between both domains can be done by the Fourier/Laplace-Transform which generally pose no problems in theory. For practical purposes however, attention should be paid to frequency and time aliasing, truncation errors etc. which will not be considered here in detail. For convention, italic symbols characterise scalar values or functions and bold symbols are vectors or matrices. Underlined symbols having an argument f are complex. They refer to the frequency domain description. Symbols with argument t or τ refer to the time domain description. If the arguments f, t or τ are omitted, the corresponding symbol or relation holds for both time and frequency domain. A dot $\cdot$ stands for a scalar product of vectors, a cross $\times$ symbolises a vector product and a star $*$

represents a convolution product. A convolution product of two vectors is to be handled like a scalar product. Furthermore, following the usual approach of S-parameter theory, all waves - guided as well as free waves - are considered by their normalised amplitudes. That is, for guided waves

the wave is travelling toward the test object:

$$a = \sqrt{Z_0}\, I^+ = V^+ / \sqrt{Z_0} \qquad [\sqrt{\mathrm{W}}], \tag{1}$$

the wave leaving the test object:

$$b = \sqrt{Z_0}\, I^- = V^- / \sqrt{Z_0} \qquad [\sqrt{\mathrm{W}}], \text{ and} \tag{2}$$

the free wave:

$$\mathbf{C} = \mathbf{E} / \sqrt{Z_s} = \sqrt{Z_s}\, \mathbf{H} \times \mathbf{e} \qquad [\sqrt{\mathrm{W/m^2}}]. \tag{3}$$

Herein I and V are the current respectively the voltage appearing at the wave guides for the corresponding wave modes. Z_0 is the characteristic impedance of the wave guide (usually 50 Ω for coax cable). **E** and **H** are the electric respectively the magnetic field vector. **e** is the unity vector of the propagation direction and $Z_s = \sqrt{\mu / \varepsilon} = (120\pi\Omega \text{ for free space})$ represents the wave impedance of the propagation medium. Note, that the polarisation of the normalised free wave **C** is identical with that of the electrical field **E**.

Finally, it shall be mentioned here that the only quantities which are accessible to a measurement are (the integral values a and b of) the guided waves whereas the free waves can not be directly measured. Thus statements about free waves can only be given via an appropriate model of the transformation device between guided and free wave i.e. the antenna. Such a device is simply a geometrical body which is able to transform guided waves into freely propagating electromagnetic waves and vice versa.

Supposing the following scenario (see Figure 1) which depicts the transmission between two arbitrary positioned antennas. Antenna 1 is fed for example by a certain signal a_1. The question is, what is the signal b_2 captured by antenna 2 which can be measured?

The antenna location in space is fixed by the position of a reference point ($\mathbf{r}_1$ or $\mathbf{r}_2$ for example) and two (preferably orthogonal) reference directions (**u** and **v**). In general these things can be chosen arbitrarily. Usually one chooses distinct geometrical elements like edges, corners, holes etc. to fix reference directions and a reference point. If this always is a good choice is questionable. But since no other criteria are applicable for the moment, there is no other way for their selection. It should be noted, that all antenna properties introduced in the following will refer to these local coordinates (**r**, **u**, **v**).

Let us now consider the antenna behaviour in a very formal way. For that purpose, it shall be distinguished between the transmission and reception mode of an antenna.

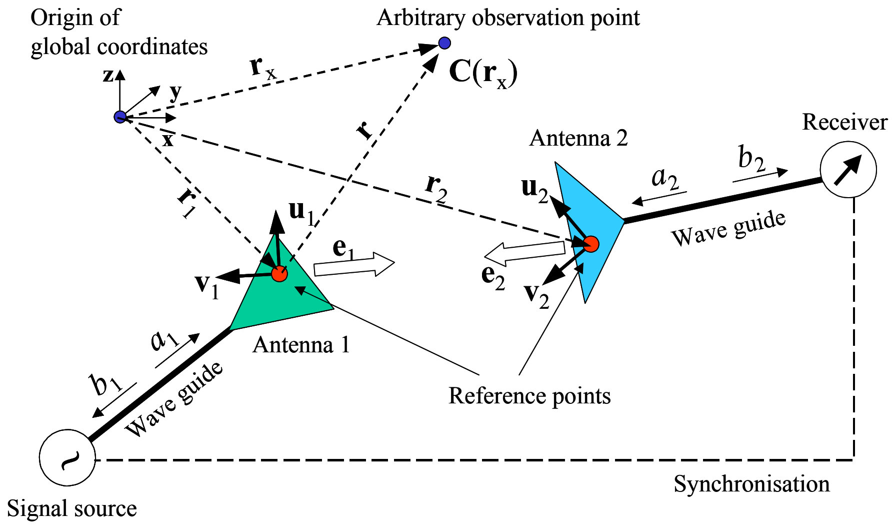

Figure 1 General set-up for transmission between two antennas. The antenna reference points and directions are fixed outside any symmetry of the antenna symbols in order to underline the arbitrariness of their choice. The distance between the reference points of both antennas is r_0. It shall be noted that the ability to measure phase depends on the measurement of the time of flight and ranges. It requires a proper synchronisation between signal source and receiver.

Transmit Mode

Supposing antenna 1 is stimulated by an incoming guided wave a_1. This results in an electromagnetic field **C** that can be observed for example at the point $\mathbf{r}_x$. That is, an antenna maps a scalar quantity (guided wave) onto a vector quantity (free wave). If the following conditions hold (which is generally the case)

- the antenna behaves linearly and time invariantly,
- the field propagates freely in all directions (isotropic propagation medium), and
- the observation point $\mathbf{r}_x$ is sufficient far away from the radiating structure (thus the radial field components are negligible),

the created field $\mathbf{C}(\mathbf{r}_x)$ can be formally expressed by:

$$\underline{\mathbf{C}}(f,\mathbf{r}_x) = \underline{a}_1(f)\underline{\mathbf{T}}_1(f,\mathbf{e})\frac{\exp - j2\pi f\tau}{r}$$
$$\updownarrow \text{Fourier Transform} \qquad (4)$$
$$\mathbf{C}(t,\mathbf{r}_x) = \frac{a_1(t-\tau) * \mathbf{T}_1(t,\mathbf{e})}{r}$$

with

distance antenna observation point $r = |\mathbf{r}_x - \mathbf{r}_1|$,

propagation direction $\mathbf{e} = \frac{\mathbf{r}_x - \mathbf{r}_1}{|\mathbf{r}_x - \mathbf{r}_1|}$,

time of flight (c – propagation speed) $\tau = r/c$, and

the antenna transfer characteristic for the transmit mode $\underline{\mathbf{T}}(f,\mathbf{e})$ or $\mathbf{T}(t,\mathbf{e})$.

Due to the transverse nature of electromagnetic waves, it furthermore holds

$$\mathbf{C} \cdot \mathbf{e} = \mathbf{T} \cdot \mathbf{e} = 0 . \tag{5}$$

Note, equ. (4)[1] represents a linear relationship between the finite propagation speed with respect to the delay term τ, the decreasing strength of the wave due to their spatial expansion is respected by $1/r$ and the behaviour of the antenna is summarised by $\underline{\mathbf{T}}(f, \mathbf{e})$ respectively $\mathbf{T}(t, \mathbf{e})$. These transfer functions are in what we are interested in. $\underline{\mathbf{T}}(f, \mathbf{e})$ is the Frequency Response Function (FRF) of the antenna in transmit mode. It is dimensionless. $\mathbf{T}(t, \mathbf{e})$ is the Impulse Response Function (IRF) of the antenna in transmit mode. Its dimension is [1/s] due to the convolution operation. The argument **e** denotes that the radiation characteristic depends on the propagation direction which is well known from classical antenna theory. As further to be seen, the characteristic antenna functions are vectors. That is, their components describe the polarisation behaviour of the radiated waves. By using a spherical coordinate system, the antenna transmit function can by expressed by two components $\mathbf{T}(\mathbf{e}) = \left[T_\varphi(\mathbf{e}) \quad T_\vartheta(\mathbf{e}) \right]$. If the ratio between both components is independent of time or frequency, the radiated wave is linear polarised.

Receive Mode

Formally, the receive antenna is doing nothing else than mapping a vector quantity (the incoming field) to a scalar quantity i.e. a guided wave leaving the antenna via the wave guide. Mathematically, this can expressed as a scalar product of two vectors. Considering antenna 2 as receiver, this results for example in the relation

$$\begin{gathered} \underline{b}_2(f) = \underline{\mathbf{R}}_2(f, \mathbf{e}) \cdot \underline{\mathbf{C}}(f, \mathbf{r}_2) \\ \updownarrow \text{Fourier Transform} \\ b_2(t) = \mathbf{R}_2(t, \mathbf{e}) * \mathbf{C}(t, \mathbf{r}_2) \end{gathered} \tag{6}$$

Herein $\underline{\mathbf{R}}(f, \mathbf{e})$ is the receive FRF having the dimension [m] and $\mathbf{R}(t, \mathbf{e})$ is the receive IRF having the dimension [m/s]. The dimension of the receive function in the time domain looks a little bit strange. But it is to be seen in conjunction with the convolution which involves additionally a time quantity. **e** is the direction of incidence. Equ. (6) supposes that the incident wave has a planar propagation front within the geometrical dimensions of the antenna.

Transmission between two antennas

Referring to (4) and (6), the receive signal b_2 can be calculated easily if the transmit and receive functions of both antennas are known. Using the scenario shown in Figure 1, it results in:

[1] Fields as represented by equ. (4) are usually called spherical waves. This is a somewhat misleading term because the (propagating) front of the wave must not exactly coincident with the surface of a sphere. Such a case appears for example if the group delay of $\underline{\mathbf{T}}_1(f, \mathbf{e})$ varies with the propagation direction **e**.

$$
\begin{aligned}
\underline{b}_2(f)\Big|_{a_2=0} &= \underline{\mathbf{R}}_2(f,\mathbf{e}_1)\cdot\underline{\mathbf{T}}_1(f,\mathbf{e}_1)\frac{\exp j2\pi f\tau_0}{r_0}\,\underline{a}_1(f) \\
&\updownarrow \text{ Fourier Transform} \\
b_2(t)\Big|_{a_2=0} &= \frac{\mathbf{R}_2(t,\mathbf{e}_1)*\mathbf{T}_1(t,\mathbf{e}_1)}{r_0} * a_1(t-\tau_0)
\end{aligned}
\tag{7}
$$

with $r_0 = |\mathbf{r}_2 - \mathbf{r}_1|$ and $\tau_0 = r_0 / c$.

$\mathbf{e}_1$ states the direction from antenna 1 to antenna 2. Note for the frequency domain formula, that the ratio between the waves $\underline{b}_2(f)$ and $\underline{a}_1(f)$ corresponds to the $\underline{S}_{21}(f)$ parameter of the whole arrangement, which can be measured by a network analyser. Correspondingly a time domain S-parameter can be defined:

$$
b_2(t)\Big|_{a_2=0} = S_{21}(t, r_0, \mathbf{e}_1) * a_1(t) \tag{8}
$$

which results in

$$
S_{21}(t, r_0, \mathbf{e}_1) = \frac{\mathbf{R}_2(t,\mathbf{e}_1)*\mathbf{T}_1(t,\mathbf{e}_1)}{r_0} * \delta(t-\tau_0). \tag{9}
$$

The scalar product in equ. (7) indicates the known fact, that the receive signal is maximum if both receive and transmit antenna are of same polarisation. Concerning (7) it is however still unsatisfactory, that an antenna has to be characterised by two different functions. In order to overcome this disadvantage, the reciprocity of the arrangement has to be involved.

Reciprocity and (complete) Friis-formula

Reciprocity is always given for linear and isotropic media, which are supposed here. It will be introduced into (7) using an equivalent approach as in [1]. Reciprocity involves that the transmission behaviour between two antenna remains unchanged if source and load are exchanged, i.e.

$$
S_{21}(r_0, \mathbf{e}_1) = S_{12}(r_0, \mathbf{e}_2) = S_{12}(r_0, -\mathbf{e}_1). \tag{10}
$$

It should be noted, that (10) is not tied to other conditions than that mentioned above. That is, far field condition and antenna back scattering do not limit (10). Insertion of (7) respectively (9) into (10) gives:

$$
\begin{aligned}
&\underline{\mathbf{R}}_2(f,\mathbf{e}_1)\cdot\underline{\mathbf{T}}_1(f,\mathbf{e}_1) = \underline{\mathbf{R}}_1(f,\mathbf{e}_2)\cdot\underline{\mathbf{T}}_2(f,\mathbf{e}_2) \\
&\updownarrow \text{ Fourier Transform} \\
&\mathbf{R}_2(t,\mathbf{e}_1)*\mathbf{T}_1(t,\mathbf{e}_1) = \mathbf{R}_1(t,\mathbf{e}_2)*\mathbf{T}_2(t,\mathbf{e}_2)
\end{aligned}
\quad . \tag{11}
$$

Equ. (11) represents a general relation which is not bounded by a specific type of antenna. That is, if it is possible to find for an arbitrary antenna an analytic relation between its transmit and receive functions, equ. (11) can be used to establish a general self-reciprocity relation which is valid for all antennas. The behaviour of a short dipole or a small loop antenna may be expressed by analytic functions, which degenerates to simple relations under far field conditions. By replacing for example antenna 2 in (11) by the receive and transmit functions of a dipole, it results the general self-reciprocity relation for any type of antennas (see annex 1 for details):

$$\underline{\mathbf{T}}(f,\mathbf{e}) = -\frac{\underline{\mathbf{R}}(f,-\mathbf{e})\cdot\mathbf{D}}{\mathrm{j}\lambda}$$

$$\updownarrow \text{ Fourier Transform} \qquad (12)$$

$$\mathbf{T}(t,\mathbf{e}) = \frac{\mathbf{R}(t,-\mathbf{e})\cdot\mathbf{D}}{2\pi c} * \frac{\partial}{\partial t}\cdots$$

Relation (12) contains a twofold vector product, which is expressed by the matrix **D** here (see also annex 1).

Introducing (12) into equ. (7) leads to the final form of the Friis-formula which describes the antennas exclusively by their receive functions (or transmit functions):

$$\begin{aligned}\underline{b}_2(f)\Big|_{a_2=0} &= -\underline{\mathbf{R}}_2(f,\mathbf{e}_1)\cdot\mathbf{D}\,\underline{\mathbf{R}}_1(f,-\mathbf{e}_1)\frac{\exp j2\pi f\tau_0}{\mathrm{j}\lambda r_0}\,\underline{a}_1(f)\\ &= -\underline{\mathbf{R}}_2(f,\mathbf{e}_1)\cdot\mathbf{e}_1\times(\mathbf{e}_1\times\underline{\mathbf{R}}_1(f,-\mathbf{e}_1))\frac{\exp j2\pi f\tau_0}{\mathrm{j}\lambda r_0}\,\underline{a}_1(f)\end{aligned}$$

$$\updownarrow \text{ Fourier Transform} \qquad (13)$$

$$\begin{aligned}b_2(t)\Big|_{a_2=0} &= \frac{1}{2\pi c r_0}\mathbf{R}_2(t,\mathbf{e}_1)*\mathbf{D}\,\mathbf{R}_1(t,-\mathbf{e}_1)*\frac{\partial a_1(t-\tau_0)}{\partial t}\\ &\frac{1}{2\pi c r_0}\mathbf{R}_2(t,\mathbf{e}_1)*\mathbf{e}_1\times(\mathbf{e}_1\times\mathbf{R}_1(t,-\mathbf{e}_1))*\frac{\partial a_1(t-\tau_0)}{\partial t}\end{aligned}$$

Equ. (13) forms the foundation for further considerations. It describes the complex frequency domain behaviour respectively the time domain behaviour of any antenna. The functions characterising the properties of an antenna are either the transmit or receive function **T**(…), **R**(…) which are given either in the frequency or time domain. Their vector nature preserves the polarisation of the electromagnetic wave.

In what follows, these functions shall be considered in more detail specifically concerning Impulse radiating Antennas.

IMPULSE RADTING ANTENNA FOR UWB-SENSING

In order to consider the antenna properties summarised by **R**(…) or **T**(…) in an appropriate way, it is advisable to start with a survey of what is expected from an (ideal) IRA applied in UWB-sensing.

Expected IRA-Properties

A typical UWB task which involves IRA's covers all variants of high resolution short range radar (Surface Penetrating Radar, Trough Wall Radar, level meter etc.) as well as (indoor) positioning systems. The basic information gained by these systems covers the time of flight and the temporal behaviour of back scattered signals which is used for target recognition. What does it mean for properties of an useful antenna?

- *Target recognition*: The only information about a target which can be gained by a radar is coded in its scattering IRF. But as seen from the radar equation (see annex 2) the target IRF is masked by the IRF of the antennas. That is, at a first glance , the antenna IRF **R**(t, **e**) should be as short as possible in order to minimise this veiling. A detailed analysis however shows, that intentionally it is sufficient to have available a wideband antenna of an arbitrary but known temporal behaviour because by impulse compression respectively de-convolution techniques the antenna influence can be (partially) eliminated from the measurement data. Modern UWB concepts (see [4] for example) apply digital impulse compression techniques so that no significant additional burden on the electronics occurs. The difficulty is however to have available an antenna whose IRF remains unchanged within the whole illumination sector of interest. Otherwise a spatial de-convolution must follow, which usually overloads current processing units. Thus, only IRA concepts remain since they best approach to a uniform radiation pattern concerning the shape of the radiated waves (within a given sector of space).

- *Time of flight*: The radar receiver determines usually the time of flight by maximum detection of the receive signal. Thus, the more pointed the signals are the more precise the measurements will be, i.e. a short antenna IRF is required. (It should be noted, that also under these circumstances it would be theoretically sufficient to suppose an arbitrary wideband antenna. But this will meet with some practical constraints as mentioned above.) But one should bear in mind that the actual desired information is a measure of distance and it is fixed by the position of two points in space. This implies that both the transmit and receive antennas have to be seen as point objects. Therefore all properties of an antenna must be attributed to an appropriate point – called centre of radiation in what follows - which is a-priory unknown. Furthermore the time of flight only refers to the time needed to propagate between the radiation centres of antenna 1 and 2. The actual measured time however still includes additional delays (which are also a-priory unknown) because the radiation centres do not coincide with the measurement planes due to the finite dimensions of the antennas.

In summarising the basic properties of an ideal antenna for UWB-sensing applications, four points should be emphasised:

1. The antenna has to radiate over a large bandwidth – a few hundred MHz to several GHz – in which the centre frequency is often within the same range as the bandwidth.
2. The radiation pours out of a concentrated centre.
3. The shape of the radiated waveform is independent from the angle of radiation within the required illumination sector. Outside this sector the radiated power should be zero.
4. The antenna IRF has to be impulse like, since otherwise point 1. to 3. can not be practically realised.

IRA model

The vector receive IRF $\mathbf{R}(t, \mathbf{e})$ or its counterpart the transmit IRF $\mathbf{T}(t, \mathbf{e})$ characterises completely the antenna behaviour concerning its properties to transform guided waves into free waves and vice versa. The foundation of their determination is given by (13) and the schematics of Figure 1.

Supposing antenna 1 is under investigation, it is rotated around its (arbitrary chosen) reference point $\mathbf{r}_1$ while measuring the transfer function in time $\mathbf{S}_{21}(t,\mathbf{e}_1)=\mathbf{S}_{12}(t,-\mathbf{e}_1)$ or frequency domain $\underline{\mathbf{S}}_{21}(f,\mathbf{e}_1)=\underline{\mathbf{S}}_{12}(f,-\mathbf{e}_1)$ [2]. From that, the desired antenna behaviour is determined by de-convolving antenna 2 which absolutely assumes the knowledge of its (boresight) impulse behaviour. There are three ways to solve this task – using a reference antenna, using two identical antennas or applying a three antenna approach. Up to this stage, the resulting antenna characteristics (for example $\mathbf{R}(t,\mathbf{e})$) refers to the centre of rotation $\mathbf{r}_1$. This is somewhat arbitrary and unhandy. What is needed are some characteristic parameters which allow comparisons between different types of antennas and parameterisation of the application software of the UWB-sensor in the easiest way.

In what follows, a general method how to define such parameters and how to acquire them will be explained. For simplicity and clarity, only a simple example shall be stressed. That is, both antenna under test and reference antenna shall be vertically polarised and only the characteristics within the horizontal plane shall be determined. For that purpose, the antenna under test is mounted on a turntable and the transmission to a reference antenna is measured. Figure 2 depicts a corresponding measurement set-up.

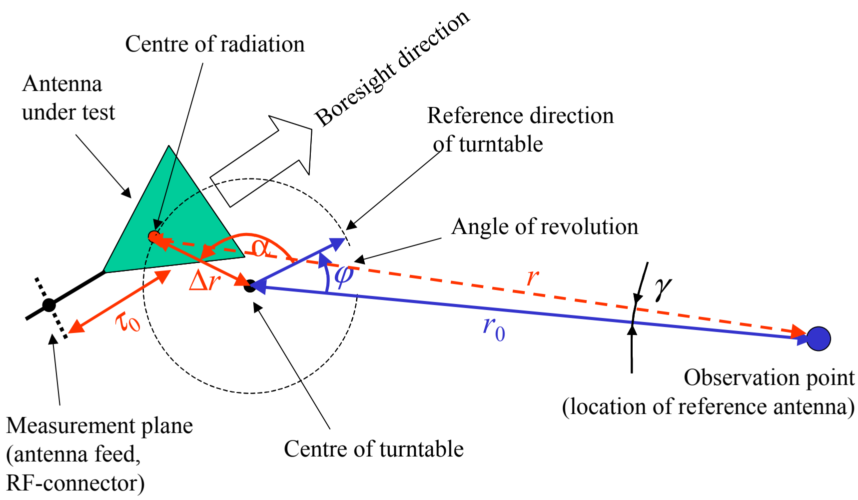

Figure 2 Measurement scenario. The antenna under test is fixed on a turntable. The antenna will generally be placed eccentrically because the centre of radiation is still unknown. These deviations are marked by Δr and α. Furthermore, the actual measurement signal refers to the measurement plane, thus the signals are subjected to an additional delay of τ_0. Note, that the centre of the turntable is identical with the antenna reference point as marked in Figure 1.

In order to prevent arbitrariness, the first step is to define a natural reference point for the antenna. Such a point is the radiation centre. It is important for IRA measurements to have such a point, because one needs a reference for phase and time measurements. Next is to define some parameters and functions which describe the antenna behaviour. This can be roughly done in concordance with the classical (power) approach but having always in mind that there are complex values for measurements with the network analyser respectively time function by using TDR (Time Domain Reflectometry) or wideband correlation. The article will introduce some definitions of characteristic

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[2] Sometimes, as for surface penetrating radar for example, it is not possible to freely dispose over the antenna of investigation. Then the radar equation (see annex 2) can be used in an adapted way to determine the antenna behaviour. One applies often simple scatterer for reference objects as spheres, wires and metal sheets. An example of such an approach is demonstrated in [5].

values for radiation in the time domain. But it will be restricted to a few examples only to show the basic procedure.

For the arrangement as depicted in Figure 2, the scattering parameters results to (compare also (7), (8), (9) and (13)):

$$
\begin{aligned}
\underline{S}_{21}(f,\varphi) &= -\underline{R}_{02}(f)\,\underline{R}_1(f,\varphi)\frac{\exp j2\pi f\tau}{\mathrm{j}\lambda r} \\
&= -\mathrm{j}\lambda T_{02}(f)T_1(f,\varphi)\frac{\exp j2\pi f\tau}{r} \\
&\updownarrow \text{ Fourier Transform} \\
S_{21}(t,\varphi) &= \frac{1}{2\pi c r}R_{02}(t)*R_1(t,\varphi_1)*\delta(t-\tau)*\frac{\partial}{\partial t}\cdots \\
&= \frac{2\pi c}{r}T_{02}(t)*T_1(t,\varphi_1)*\delta(t-\tau)*\int_{-\infty}^{t}\cdots \mathrm{d}\xi
\end{aligned} \quad . \qquad (14)
$$

Here, antenna 2 has been supposed to be the reference antenna. Antenna 1 is the antenna under test. As long as the angle γ remains small during the measurements, it would be sufficient to deal with the boresight characteristic R_{02} respectively T_{02} of the reference antenna. In order to show the equivalence between transmit $T(\ldots)$ and receive functions $R(\ldots)$, equ. (14) is given for both variants. Equ. (14) supposes, that the distance between both antennas is large enough to respect the far field condition as well as to restrict the angle γ to small values.

The Centre of Radiation

The exact radiation centre of an antenna is usually not known a-priory. That's why it is difficult to fix the antenna onto the turntable in such a way that radiation point and rotation axis coincide. Supposing first an omini-directional IRA which has a pronounced radiation centre, the measured time of flight τ will vary sinusoidal around the mean value $\tau_{pr}+\tau_0 = r_0/c+\tau_0$ by rotating the turntable (see for demonstration the measurement examples in Figure 4). Having registered that, it is an easy task to gain the actual radiation centre (determined by Δr and α) and the internal delay τ_0 of the antenna which allows to centre up[3] the measurement values $S_{21}^{(meas)}(t,\varphi)$ or $\underline{S}_{21}^{(meas)}(f,\varphi)$:

$$
\begin{aligned}
\underline{S}_{21}^{(centr)}(f,\varphi) &= \frac{r}{r_0}\,\underline{S}_{21}^{(meas)}(f,\varphi)\exp \mathrm{j}2\pi f\left(\frac{r-r_0}{c}+\tau\right) \\
&\approx \underline{S}_{21}^{(meas)}(f,\varphi)\exp \mathrm{j}2\pi f\left(\frac{r-r_0}{c}+\tau\right)
\end{aligned} \quad . \qquad (15)
$$

The "centred up values" are the starting point for the parameter extraction. However it is not as easy in practice to find the radiation centre, because most antennas are neither omni-directional nor have a pronounced radiation centre. Thus, one can only fix the centre in a way that best approaches the real conditions. The usual approach is to replace the real antenna by an antenna model which provides a circular wave front (thus a centre can be defined). The "weight" of the waves "radiated" from the antenna model into a certain direction should be the same as for the real antenna in that

[3] The easiest to do this, is to deal with the frequency domain as shown in equ. (15) because this allows a continuous time shift.

direction. To do so, the antenna behaviour is globalised by a first order p-moment $m_p(\varphi)$ for all directions of radiation formed from the product of a " characteristic value" of the time of flight $\tau_{cg}(S_{21}(t,\varphi))\big|_p$ (representing a p-centre of gravity – see annex 3) and the "weight" of the radiated pulse $\|S_{21}(t,\varphi)\|_p$ (representing a p-norm – see annex 3) :

$$m_p(\varphi) = \tau_{cg}(S_{21}(t,\varphi))\big|_p \, \|S_{21}(t,\varphi)\|_p \, . \qquad (16)$$

The " characteristic value" of the time of flight refers to the p-centre of gravity of the IRF and the "weight" corresponds to the p-norm of the IRF. p may be any integer number, but only three values are of practical importance:

- $p = 1$ for the rectified IRF,
- $p = 2$ for the squared IRF (signal energy), and
- $p = \infty$ maximum value of IRF.

See annex for definition of p-norm and p-centre of gravity.

Finally, it is required that the difference between the moments of the circular antenna (model) located at the point of rotation

$$m_p^{(circ)}(\varphi) = \tau_{pr} \, \|S_{21}(t,\varphi)\|_p \qquad (17)$$

and the moments of the antenna under test which should be centred up

$$m_p^{(centr)}(\varphi) = \tau_{cg}\left(S_{21}^{centr}(t,\varphi)\right)\Big|_p \, \|S_{21}(t,\varphi)\|_p \qquad (18)$$

approaches a minimum, i.e

$$\begin{bmatrix} \Delta r \\ \alpha \\ \tau_0 \end{bmatrix} \Rightarrow \min_{\Delta r,\alpha,\tau_0} \left(\sum_{\varphi_i} \left(m_p^{(centr)}(\varphi_i) - m_p^{(circ)}(\varphi_i) \right)^2 \right). \qquad (19)$$

The values gained from (19) fix the centre of radiation (Δr, α) and give an integral value for the internal delay (τ_0) of the antenna. Note that these values may vary by applying different p-norms. The choice of a certain p-norm should be based on the actual method of signal detection and the circumstances of the measurement problem. Having determined the indicated values, the transmit $T(t, \varphi)$ respectively receive IRF's $R(t, \varphi)$ can be calculated from the centred up measurements by equ. (14).

The far field

It is critical for some application to know the minimum distance which approximately holds far field conditions. Surface Penetrating Radar is an application on short distances where this can be important. There are theoretical estimations of the minimum distance for far field conditions. But they often give very conservative values.

Measurements can be more realistic under these circumstances. Keeping the Friis-equation by varying the antenna distance r could provide such a useful criterion. Thus, a error threshold K_1 can be established which the ratio (20) should always exceed:

$$\frac{r\,\|S_{21}(t,r)\|_p}{r_{ff}\,\|S_{21}(t,r_{ff})\|_p} \geq K_1 \tag{20}$$

Herein r_{ff} is an antenna distance which surely lies in the far field zone.

Antenna pattern and beam width

In the classical sense, the antenna pattern relates the properties of a wave radiated in an arbitrary direction to the properties of a reference wave. Usually one takes the radiated spatial power density and the boresight direction for the reference curve.

For IRA's, it can be done in an equivalent way by relating IRF's. However there are different possibilities which results from the question, what is the best reference and how to relate the IRF of an arbitrary direction of radiation to the reference IRF.

Let's start by considering the reference function. Most applications of UWB-sensing do not involve a spatial de-convolution of the antenna pattern. In such cases, a "representative" FRF respectively IRF T_0 , R_0 is used to represent the radiation behaviour of the whole antenna (of course restricted within the beam width). The simplest way to choose a reference function is to use the original boresight IRF[4]:

$$R_0(t)\big|_{bs} = R_{bs}(t) = R(t,\varphi_0) \tag{21}$$

(φ_0 being the boresight direction). A second possibility consists of applying an average time shape gained from all waveforms radiated within the beam width b_w:

$$R_0(t)\big|_{av} = R_{0av}(t) = \frac{1}{b_w}\int_{b_w} R(t,\varphi)d\varphi\,. \tag{22}$$

Note that the characteristic delay of both functions i.e. its p-centre of gravity has to be fixed to τ_0.

Now, the antenna pattern Θ can be defined. Three different types shall be introduced. The notation $\Theta\big|_{type,ref}$ indicates the type and the reference function applied (*reference* = *bs* or *av*).

- Time pattern $\Theta\big|_{t,ref}$: It represents the real variation of the temporal shape of the IRF along the radiation angle. It results from a de-convolution which is preferably calculated in the frequency domain.

$$\underline{\Theta}(f,\varphi)\big|_{t,\,ref} = \frac{\underline{R}(f,\varphi)}{\underline{R}_0(f)\big|_{ref}} = \frac{\underline{S}_{21}^{(centr)}(f,\varphi)}{\underline{S}_{21,0}^{(centr)}(f)\Big|_{ref}}$$

respectively (23)

$$R(t,\varphi) = \Theta(t,\varphi)\big|_{t,\,ref} * R_0(t)\big|_{ref}$$

$$S_{21}^{(centr)}(t,\varphi) = \Theta(t,\varphi)\big|_{t,\,ref} * S_{21,0}^{(centr)}(t)\Big|_{ref}$$

---

[4] Here, the receive IRF will used to define the radiation pattern. It is however also usual to refer to the transmit function.

$S_{21,0}^{(centr)}\Big|_{ref}$ is determined according (21) or (22).

- Weight pattern $\Theta|_{w,ref,p}$: It refers to the integral "weight" of the radiation in different directions.

$$\Theta(\varphi)|_{w,ref,p} = \frac{\|R(t,\varphi)\|_p}{\left\|R_0(t)|_{ref}\right\|_p} \tag{24}$$

Note that $\Theta(\varphi)|_{w,ref,p} \neq \dfrac{\left\|S_{21}^{(centr)}(t,\varphi)\right\|_p}{\left\|S_{21,0}^{(centr)}(t)\big|_{ref}\right\|_p}$.

- Coherence pattern $\Theta|_{f,ref,p}$: It gives a global measure how the shape of the radiated wave form changes along the angle of radiation.

$$\Theta(\varphi)|_{f,ref,p} = \frac{\|R(t,\varphi) * R(-t,\varphi)\|_p}{\sqrt{\|R(t,\varphi) * R(-t,\varphi)\|_p \left\|R_0(t)|_{ref} * R_0(-t)|_{ref}\right\|_p}} \tag{25}$$

The definition of beam width b_w is finally based on (24) and (25) by fixing certain thresholds. In that sense, the beam width $b_w|_{w,p}$ represents a region of illumination in which the strength ("weight") of the radiated waves (respectively the sensitivity to receive signals) can be considered as nearly constant. In accordance, the beam width $b_w|_{f,p}$ fixes an area of radiation where the shape of radiated waves remains nearly constant, i.e. within this area a spatial de-convolution can be omitted without dramatic loss on information. It should be noted, that for generality, the definitions (24) and (25) are given in accordance to the p-norm concept. But this does not mean, that all possible norms bring useful results for any type of antenna. The infinity norm was the best choice for the example below.

Further parameters

As noted above, the functions $R_0(t)|_{type}$ respectively $T_0(t)|_{type}$ represents two characteristic types of an antenna IRF. As such they are typical impulse functions which can characterised by a lot of usual parameters. Such parameters are well known and will not be considered here. A consideration of the energy balance would bring further parameters to the light as feed point reflection, antenna efficiency or antenna back scattering.

MEASUREMENTS

Some measurement examples shall be shown for demonstration. The measurement arrangement covers a turntable to rotate the antenna under test, a reference antenna (usually of same type as the antenna under test) and a measurement device. An integrated UWB-system [4] was used for measurements up to 4 GHz bandwidth. Concerning the ability of the UWB-head (very high measurement speed), the antennas could be rotated up to several hundreds rpm. The mechanical unbalance of the antennas however limited the number of revolutions to lower values. The advantage of a high measurement speed is the robustness against temporal variations within the space of measurement (ordinary office space or laboratory). Network analysers were used if a higher bandwidth was required. But such devices greatly limit the measurement speed to the point that there should not be too much movement in the laboratory. Due to the large bandwidth, a absorber hall was not required because perturbing reflections could be gated out from the measurements.

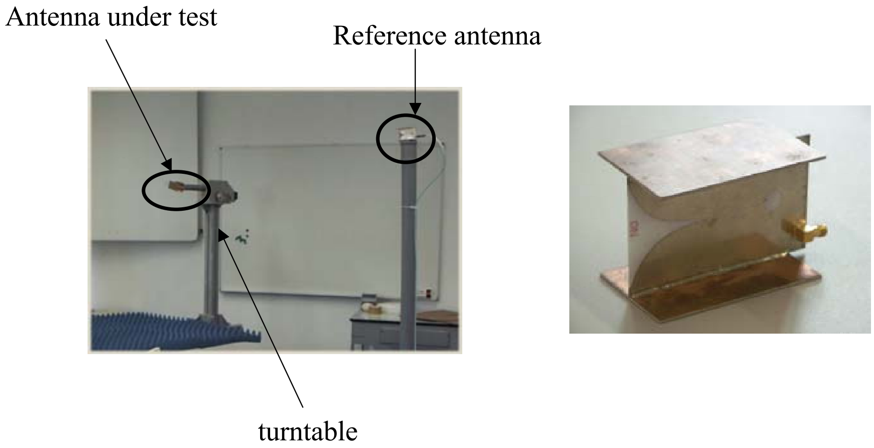

Figure 3 Example of the measurement set-up. Left: Turntable and reference antenna. Right: Vivaldi antenna as used for some measurement examples.

In order to demonstrate the effect of misalignment between the rotation axis of the turntable and the centre of radiation, the measurements of the Vivaldi antenna were repeated two times. The first measurement was done by estimating as good as possible the centre of radiation and colocating this with the axis of the turntable. In the second experiment, a misalignment of a few centimetre was introduce. Figure 4 is showing the measurement result after eliminating perturbing reflections for both experiments. The images indicate the influence of the misalignment. It is also visible, that the estimation of the centre of radiation in the first experiment was not as good since the centre of gravity curve is far from a horizontal line.

It seems that the radiation splits in two centres by turning out of boresight direction. This effect could be observed more clearly at a large TEM-horn (approximately 1 m of length) as depicted in Figure 5.

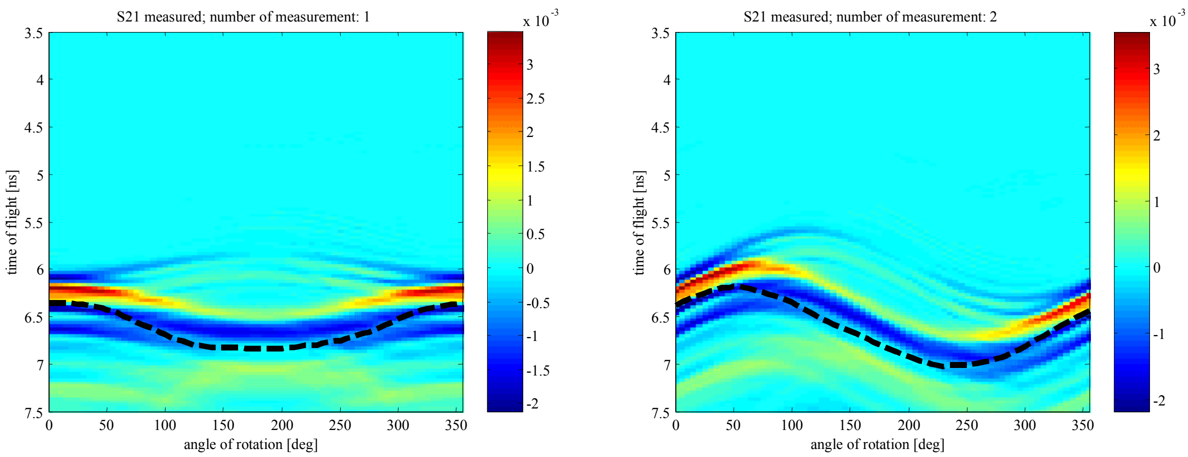

Figure 4 Measurement results for both experiments with Vivaldi antenna. The data represents a full rotation of 360°. The colour represents the measured IRF $S_{21}(t,\varphi)$. The dotted line indicates the 2-centre of gravity (i.e energy values) of all IRFs.

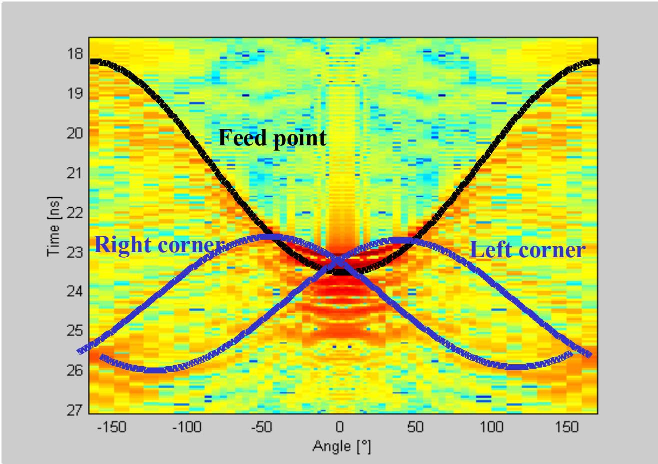

Figure 5 Measurement data for a large TEM-horn. The horn was built from a triangular sheet over a ground plane. The characteristic variations of several signal components by their delay time indicates the radiation centres. The effect of the three edges of the sheet are emphasised by black and blue lines. The amplitude values are given in a logarithmic scale. (courtesy MEODAT GmbH)

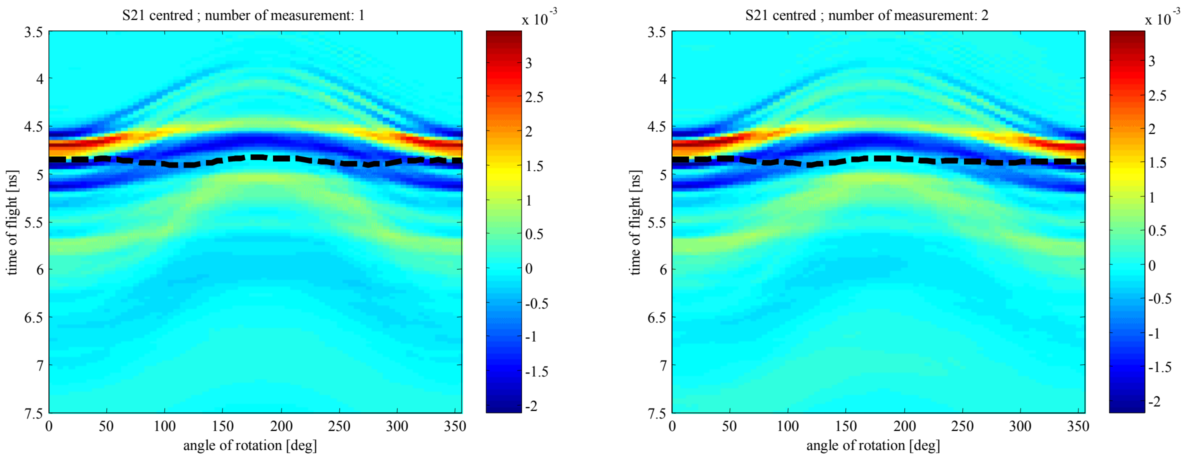

Figure 6 Centred up data for the Vivaldi antenna. Obviously the antenna does not provide a circular wave front and the radiation splits up in two centres.

The measurements from both experiments of Figure 4 were used to centre up the data (see equ. (15) to (19)). Figure 6 depicts the results. Both images are in good accordance representing the same antenna characteristic. From this data the antenna receive and transmit IRF can be analysed (compare Figure 7 and Figure 8). The 3 dB-beam width gained from the infinity-norm pattern is about -45° … 60°. The coherence within this interval is better than 85 %. As to be seen from the pattern, the used antenna sample is subjected to some asymmetries.

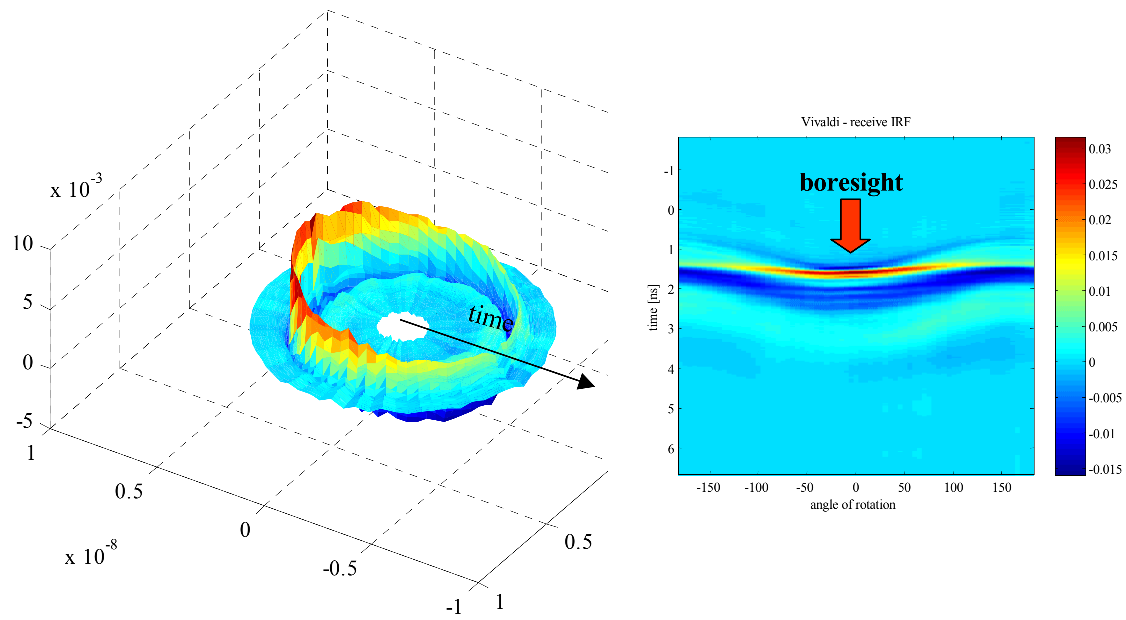

Figure 7 The receive IRF $R(t,\varphi)$

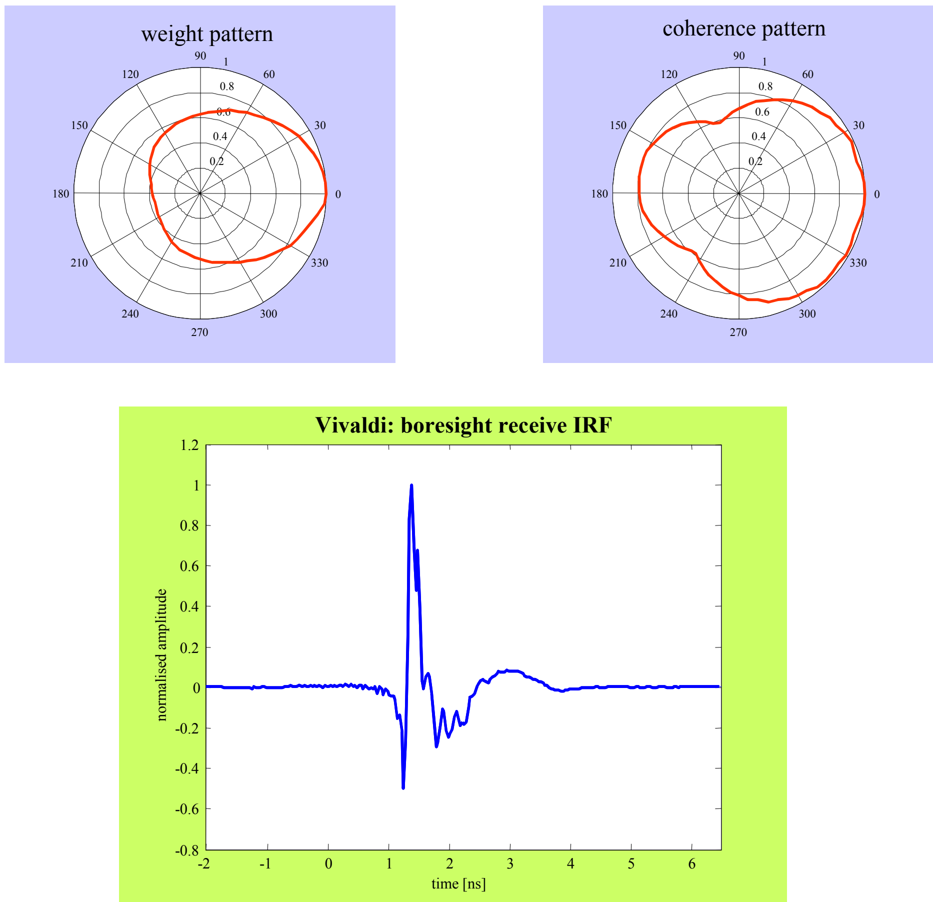

Figure 8 Infinity-norm pattern of the Vivaldi antenna and boresight IRF $R(t,\varphi_0)$

SUMMARY

Impulse radiating antennas are of great importance for high resolution UWB-sensing. For that purpose it is usual to introduce time domain parameters to characterise such antennas. A general antenna model for the complex frequency domain as well as for the time domain was developed. It forms the foundation for definition of the introduced parameters. Herein the p-norm plays an important role.

References:

[1] C.E. Baum: General Properties of Antennas. Sensor and Simulation Note 330; July 1991
[2] K. Simonyi: Theoretische Elektrotechnik. VEB Deutscher Verlag der Wissenschaften, Berlin 1989
[3] R. Zurmühl, S. Falk: Matrizen 1, Grundlagen. Springer-Verlag Berlin Heidelberg 1997
[4] J. Sachs, P. Peyerl, F. Tkac, M. Kmec: Digital ultra-wideband-sensor electronics integrated in SiGe-technology. Proceedings of the EuMC 2002; Vol. II, p. 539-542 , 23 – 27 September 2002, Milan (Italy)
[5] R. Zetik, J. Sachs, B. Schneegast: Evaluation of antenna pattern for radiation in solid media. International Radar Symposium IRS 98, Munich 15-17. September 1998

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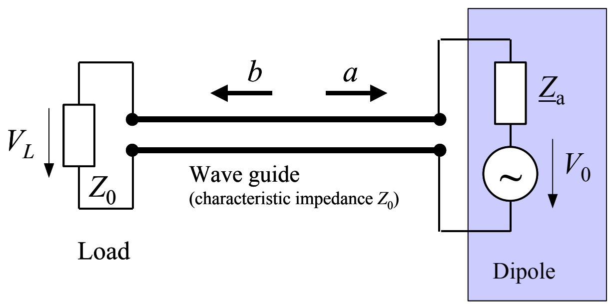
Annex 1

For simplicity the following considerations will restricted to the frequency domain. The final results will however be represented in both domains – time and frequency.

The short electrical Dipole: Receive Mode



The open circuit voltage of a short dipole is given by

$$\underline{V}_0(f) = \mathbf{h} \cdot \underline{\mathbf{E}}(f),$$

where the vector **h** is representing the length of the dipole (antenna height) and its position in space.

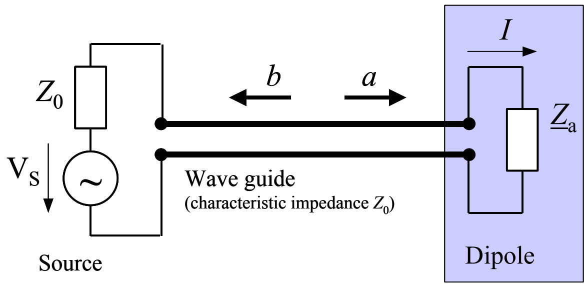
Corresponding to the electrical equivalent circuit for the receive mode, the normalised wave b results from

$$\underline{b}(f)=\frac{\underline{V}_L(f)}{\sqrt{Z_0}}=\frac{Z_0}{\sqrt{Z_0}(Z_0+\underline{Z}_a)}\underline{V}_0(f)=\frac{\sqrt{Z_s}Z_0}{\sqrt{Z_0}(Z_0+\underline{Z}_a)}\mathbf{h}\cdot\frac{\underline{\mathbf{E}}(f)}{\sqrt{Z_s}}=\sqrt{\frac{Z_s}{Z_0}}\frac{Z_0}{(Z_0+\underline{Z}_a)}\mathbf{h}\cdot\underline{\mathbf{C}}(f).$$

Thus the receive FRF of a short dipole is:

$$\mathbf{R}_D(f)=\sqrt{\frac{Z_s}{Z_0}}\frac{Z_0}{(Z_0+\underline{Z}_a)}\mathbf{h}$$

The short electrical Dipole: Transmit Mode



By [2] the radiated field of a dipole at a sufficient large distance is

$$\mathbf{E}(t,\mathbf{r})=\frac{\mu}{4\pi}\frac{1}{r}\frac{\partial I(t-\tau)}{\partial t}(\mathbf{h}\times\mathbf{e})\times\mathbf{e}\ .$$

This can also be expressed by a matrix-product in which the matrix **D** is built from the components of **e** (for details see [3])

$$\mathbf{E}(t,\mathbf{r})=\frac{\mu}{4\pi}\frac{1}{r}\frac{\partial I(t-\tau)}{\partial t}\mathbf{D}\,\mathbf{h}\,.$$

The corresponding frequency domain representation is

$$\underline{\mathbf{E}}(f,\mathbf{r})=\mathrm{j}f\frac{\mu}{2}\underline{I}(f)\frac{\exp(-\mathrm{j}2\pi f\tau)}{r}\mathbf{D}\,\mathbf{h}\,.$$

The wave a injected into the dipole calculates from

$$a=\frac{V_S}{2\sqrt{Z_0}},$$

thus the current which drives the dipole may be expressed by

$$\underline{I}(f)=\frac{2\sqrt{Z_0}}{Z_0+\underline{Z}_a}\underline{a}(f).$$

Introducing this into the radiation formula and normalising the electrical field, it finally states:

$$\underline{\mathbf{C}}(f,\mathbf{r})=\frac{\underline{\mathbf{E}}(f,\mathbf{r})}{\sqrt{Z_s}}=\mathrm{j}f\sqrt{\frac{Z_0}{Z_s}}\frac{\mu}{Z_0+Z_a}\underline{a}(f)\frac{\exp(-\mathrm{j}2\pi f\tau)}{r}\mathbf{D}\,\mathbf{h}\ ,$$

thus the transmit FRF of a dipole is

$$\underline{\mathbf{T}}_D(f,\mathbf{r})=\mathrm{j}f\sqrt{\frac{Z_0}{Z_s}}\frac{\mu}{Z_0+Z_a}\mathbf{D}\,\mathbf{h}\ .$$

Antenna self-reciprocity

Supposing the antenna 2 is the dipole and by applying $Z_s=\sqrt{\mu/\varepsilon}$, $c=1/\sqrt{\mu\varepsilon}=f\,\lambda$ as well as $\mathbf{e}_1=-\mathbf{e}_2=\mathbf{e}$, one finally gets from (11) the general reciprocity relation for any antenna in frequency and time domain:

$$\begin{gathered}\underline{\mathbf{T}}(f,\mathbf{e})=-\frac{\underline{\mathbf{R}}(f,-\mathbf{e})\cdot\mathbf{D}}{\mathrm{j}\lambda}\\ \updownarrow\ \text{Fourier Transform}\\ \mathbf{T}(t,\mathbf{e})=\frac{\mathbf{R}(t,-\mathbf{e})\cdot\mathbf{D}}{2\pi c}*\frac{\partial}{\partial t}\cdots\end{gathered}\ .$$

Multiplying the upper equation by its complex conjugate results, for boresight radiation in the well known reciprocity equation for power values like antenna gain and aperture

$$\begin{gathered}\underline{\mathbf{T}}(f)\cdot\underline{\mathbf{T}}^*(f)=\frac{\underline{\mathbf{R}}(f)\cdot\underline{\mathbf{R}}^*(f)}{\lambda^2}\\ G(f)=\frac{A_{eff}(f)}{4\pi\lambda^2}\end{gathered}\ ,$$

because the antenna (power) gain is given by definition as:

$$G(f,\mathbf{e})=\frac{\underline{\mathbf{C}}(f,\mathbf{r})\cdot\underline{\mathbf{C}}^*(f,\mathbf{r})}{\frac{1}{4\pi r^2}\underline{a}(f)\underline{a}^*(f)},$$

which leads to:

$$G(f,\mathbf{e})=4\pi\,\underline{\mathbf{T}}(f,\mathbf{e})\cdot\underline{\mathbf{T}}^*(f,\mathbf{e}).$$

The dimension of $\underline{\mathbf{R}}(f)$ is [m], consequently its square has the dimension of an area. Thus one attributes an area to the antenna – called as effective aperture - over which the antenna shall capture the wave energy even if its geometrical shape is not dominated by any area:

$$A_{eff}(f)=\underline{\mathbf{R}}(f)\cdot\underline{\mathbf{R}}^*(f).$$

Annex 2

Time domain Radar equation

The radar equation can be developed in the same formal way as the Friis-Formula. One has only to insert the effect of the scatterer into the transmission path. A scatterer provides a spherical wave (if seen from distance) caused by an incident field. If the incident field is considered as locally planar within the dimensions of the scatterer, the scattering effect is nothing else than a mapping of one vector quantity to an other. This is expressed by the matrix relation

$$\underline{\mathbf{C}}_{scatterd}(f,\mathbf{e}_2)=\underline{\mathbf{S}}_{sc}(f,\mathbf{e}_1,\mathbf{e}_2)\cdot\underline{\mathbf{C}}_{incident}(f,\mathbf{e}_1)\frac{\exp \mathrm{j}2\pi\tau}{r}$$

$\updownarrow$ Fourier Transform

$$\mathbf{C}_{scatterd}(t,\mathbf{e}_2)=\frac{\mathbf{S}_{sc}(t,\mathbf{e}_1,\mathbf{e}_2)*\mathbf{C}_{incident}(t-\tau,\mathbf{e}_1)}{r}$$

in which $\mathbf{S}_{sc}(\mathbf{e}_1,\mathbf{e}_2)$ is the scattering IRF respectively FRF matrix of the target. It depends on the direction of incident $\mathbf{e}_1$ as well as on the direction of observation $\mathbf{e}_2$. $\mathbf{r}$ refers to the distance between scatterer and observation point. τ is the corresponding propagation time.

Combining this with the transmit antenna (see equ. (4)), the receive antenna (see equ.(6)) and the reciprocity relation (see annex 1), the radar equation becomes:

$$\underline{b}_2(f)\big|_{a_2=0}=-\underline{\mathbf{R}}_2(f,\mathbf{e}_2)\cdot\underline{\mathbf{S}}_{sc}(f,\mathbf{e}_1,\mathbf{e}_2)\cdot\mathbf{e}_1\times(\mathbf{e}_1\times\underline{\mathbf{R}}_1(f,-\mathbf{e}_1))\frac{\exp j2\pi f(\tau_1+\tau_2)}{\mathrm{j}\lambda r_1 r_2}\underline{a}_1(f)$$

$\updownarrow$ Fourier Transform

$$b_2(t)\big|_{a_2=0}=\frac{1}{2\pi c r_1 r_2}\mathbf{R}_2(t,\mathbf{e}_1)*\mathbf{S}_{sc}(t,\mathbf{e}_1,\mathbf{e}_2)*\mathbf{e}_1\times(\mathbf{e}_1\times\mathbf{R}_1(t,-\mathbf{e}_1))*\frac{\partial a_1(t-\tau_1-\tau_2)}{\partial t}.$$

Herein r_1, τ_1 refers to the distance/time of flight between transmit antenna and scatterer. Correspondingly, r_2, τ_2 belongs to scatterer – receive antenna.

Annex 3

Norm and centre of gravity

The norm of a function is a (single value) parameter which intends to give a measure of its "weight" or "size". There are lots of possibilities to introduce such a value. The p-Norm is an often used definition:

$$\|g(t)\|_p=\sqrt[p]{\int_{-\infty}^{\infty}|g(t)|^p\,\mathrm{d}t}$$

As can be seen, there exists an infinite number of p-norms. But only 4 are of practical importance:

$p = 1$ The 1-norm represents the area under the “rectified” function.

$p = 2$ The 2-norm represents the area under the squared function, i.e. it is a measure of total energy or power.

$p = \infty$ The ∞-norm gives the magnitude (maximum absolute value) of the function.

$p = -\infty$ The -∞-norm gives the minimum (absolute) value of the function. This will however not applied for the topics discussed here.

A useful extension of the *p*-norm to a vector function $\mathbf{g}(t) = [g_1(t) \quad g_2(t) \quad g_3(t)]^T$ could be to replace $g(t)$ in the equation above by $g(t) = |\mathbf{g}(t)| = \sqrt{g_1^2(t) + g_2^2(t) + g_3^2(t)}$, i.e. with the 2-norm of its components.

The *p*-centre of gravity of a function can be defined in a corresponding way

$$\tau_{cg}(g)\Big|_p = \frac{\sqrt[p]{\int_{-\infty}^{\infty} t\,|g(t)|^p \,\mathrm{d}t}}{\sqrt[p]{\int_{-\infty}^{\infty} |g(t)|^p \,\mathrm{d}t}} \qquad p \geq 1 .$$

The physical meaning is in accordance with the list above. The -∞-centre of gravity is however senseless.